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SYSTEM IDENTIFICATION FOR LARGE SPACE STRUCTURE DAMAGE ASSESSMENT

J. C. CHEN

J. A. GARBA

Applied Technologies Section
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

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It becomes clear that continuous monitoring of structural integrity and the detection/assessment of damage and its subsequent compensation, repair, and control are important considerations for large flexible space structures. The ultimate future requirement will be a remote data acquisition system with rapid on board analysis for almost real time assessment. The present study will focus on the feasibility of such a methodology which will provide a damage assessment capability for large space structures.

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BACKGROUND

- Early warnings for structural damage are necessary for prevention of catastrophic failure.

- For safety, the damage should be continuously monitored as to its occurrence, its location and as to the extend of the damage.

- Certain damage such as the material degradation cannot be detected by visual inspection.

- Damage detection permits real time corrective action to minimize further damages and to maintain operational requirement.

Techniques of using experimentally measured data for determining the parameters in the equations of motion of a system is commonly called system identification. A typical procedure involves the modal test of the structural system during which the responses due to external excitations are measure. From the response data, the dynamic characteristics of the system such as the natural frequencies and mode shapes can be determined directly or through data processing techniques depending on the test method employed. Because the natural frequencies and mode shapes of a structural system are functions of the system parameters such as the mass and stiffness, these system parameters may be "identified" by comparing those determined by test to those dynamic characteristics predicted from the mathematical model.

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TECHNICAL APPROACH

Objective:

- Detect Occurrence
- Identify Location
- Quantity Damage

Methodology:

- Comparison of dynamic characteristics between the healthy state and damaged state.
- Identification procedure for model changes.
- Relating model changes to damages.

The fundamental questions for damage assessment are whether it is feasible to identify the occurrence, location and extend of the damage from given measured structural dynamic characteristics. Therefore, the relationship between the physical parameters such as the mass and stiffness and the dynamic characteristics such as the eigenvalues and eigenvectors or natural frequency and mode shape must be established.

It is clear that values of ω_i and $\{\phi\}_i$ are functions of the mass [M] and stiffness [K] of the system. In other words, any changes in [M] and [K] due to the loss of mass or loss of stiffness of certain parts of the structural system will be reflected in its natural frequency and mode shape measurements. A discovery of a deviation of the measured natural frequency and mode shape with respect to those previously measured when the system was in an undamaged condition is an indication of the occurrence of damage. Certain changes in the physical parameters will effect certain modes but not others.

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GOVERNING EQUATION FOR STRUCTURES

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\}$$

Eigensolution:

$$[K]\{\phi\}_i - \omega_i^2 [M]\{\phi\}_i = 0$$

$\{\phi\}_i$, ω_i are function of $[M]$, $[K]$

Any changes of $\{\phi\}_i$ and ω_i are indications of occurrence.

First, it will be postulated that the mass distribution of the system [M] remains unchanged or is changed by only a known quantity. This is a reasonable assumption in that most structural damage for the large space structures will result in stiffness losses instead of complete separation or breakage with a loss of mass. Also for certain large space structures, such as the space station, the major distribution to the mass matrix came from non-load carrying components such as instrument packages, fluid containers and power generation units. These weights can be accurately estimated.

The kinetic energy distribution at each degree-of-freedom for the ith mode is equal to the potential energy distribution. The potential energy distribution is a function only of the stiffness matrix and the modal displacements. For a localized damage, the mode shape should be similar to or only slightly deviated from that of the undamaged system. Hence the potential energy distribution will be similar except for those degrees-of-freedom associated with the damaged component. The location of the damage can be found by identifying those degrees-of-freedom whose kinetic energies are different from those of the undamaged system. Since the correct stiffness matrix for the damaged system is unknown, the kinetic energy distribution can be used for this purpose instead.

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KINETIC AND POTENTIAL ENERGY

$$\lceil \phi_i \rfloor [K] \{\phi\}_i = \omega_i^2 \lceil \phi_i \rfloor [M] \{\phi\}_i$$

$$\{\phi_{ij} (K_{jk} \phi_{ik})\} = \omega_i^2 \{\phi_{ij} (M_{jk} \phi_{ik})\}$$

i = mode number,

j = DOF number, sum over k = 1, 2, 3

For diagonal mass matrix j = k

$$\{\phi_{ij} (K_{jk} \phi_{ik})\} = \omega_i^2 \{M_{jj} \phi_{ij}^2\}$$

- Assume [M] remain unchanged.
- Damage causes small changes in mode shapes but substantial changes in local potential energy.

It should be noted that the dimensions of the vectors $\{y\}_i$ and $\{\Delta k_{ij}\}$ are different. The dimension of $\{y\}_i$ is equal to the number of degrees-of-freedom of the system and the dimension of $\{\Delta k_{ij}\}$ will be equal to the number of independent elements in the stiffness matrix. Although the stiffness matrix is highly banded, the number of independent elements is usually larger than the number of the degrees-of-freedom of the system.

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QUANTIFICATION OF DAMAGE

Decomposition of Stiffness Matrix

[K] = [Ko] + [ΔK]

[Ko] = Stiffness for undamaged system

[ΔK] = Perturbation stiffness due to damage.

GOVERNING EQUATION

$$[\Delta K]\{\phi\}_i = \left(\omega_i^2 [M] - [K_o]\right)\{\phi\}_i = \{\gamma\}_i$$

$$[C]_i \{\Delta k_{ij}\} = \{\gamma\}_i$$

$[C]_i$ = Connectivity matrix, function of

Δk_{ij} = Elements in [ΔK] matrix, representing damages as reduction of stiffness in the associated structural elements.

For using multiple modes, the equation can be obtained by stacking the single mode equations. The number of unknowns remains the same, however, the number of equqtions will be increased. It is possible that the number of equations may become larger than that of the unknowns. In principle, a solution will not exist for such case, but an approximation can be obtained.

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QUANTIFICATION OF DAMAGE (CONTINUE)

For multiple modes

$$\begin{bmatrix} [C]_1 \\ [C]_2 \\ \vdots \\ [C]_n \end{bmatrix} \{\Delta k_{ij}\} = \begin{Bmatrix} \{y\}_1 \\ \{y\}_2 \\ \vdots \\ \{y\}_n \end{Bmatrix}$$

or

$$\underset{n \times m}{[C]} \; \underset{m \times 1}{\{\Delta k_{ij}\}} = \underset{n \times 1}{\{Y\}}$$

n = number of equations = number of modes x DOF

m = number of Δk_{ij}'s, the unknown

in general $n \neq m$

Due to the nature of the problem, namely to seek the changes in the stiffness structural elements as represented by the quantities Δk_{ij}, it is reasonable to postulate that the "optimal" solution is the one with the smallest Euclidian norm. Thus, the solution procedure becomes a constrained minimization

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QUANTIFICATION OF DAMAGE (CONTINUE)

Case I $\quad m > n \quad$ (more unknowns than equations)

- Infinite number of solution.
- Seeking one with minimum Euclidean norm.

$$E = \frac{1}{2} \| \Delta k_{ij} \| + \{\lambda\}^T \left(\{\Upsilon\} - [C]\{\Delta k_{ij}\} \right)$$

where $\| \Delta k_{ij} \| = \{\Delta k_{ij}\}^T \{\Delta k_{ij}\}$ Euclidean Norm

$\{\lambda\}$ = vector of Lagrange multipliers

$$\left\{ \frac{\partial E}{\partial (\Delta k_{ij})} \right\} = 0 \quad \text{AND} \quad \left\{ \frac{\partial E}{\partial \lambda} \right\} = 0$$

$$\{\Delta k_{ij}\} = [C]^T \left([C][C]^T \right)^{-1} \{\Upsilon\}$$

If number of equations is greater than the number of unknowns, no exact solution exists. Therefore, one may look for an approximate solution in which the Euclidian norm of the error is minimized.

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QUANTIFICATION OF DAMAGE (CONTINUE)

Case II $n > m$ (more equations than unknowns)

- No solution
- Seeking approximation with minimum error

$$\{\epsilon\} = \{\Upsilon\} - [C]\{\Delta k_{ij}\}$$

$$\|\epsilon^2\| = \{\epsilon\}^T\{\epsilon\}$$

$$\left\{\frac{\partial\|\epsilon^2\|}{\partial(\Delta k_{ij})}\right\} = 0$$

$$\{\Delta k_{ij}\} = \left([C]^T[C]\right)^{-1}[C]^T\{\Upsilon\}$$

ILLUSTRATIVE EXAMPLE

A simple example will be used for the purpose of illustrating the proposed approach for the assessment of structural damage.

Figure 1 shows a 3 degree-of-freedom system to be used as an example for both the undamaged and damaged configurations. The "damage" is assumed to be the complete breakage of the spring K_4. The mass and stiffness matrices for the undamaged configuration are as follows:

$$[M] = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} , \qquad [K] = \begin{bmatrix} 6 & -3 & 0 \\ -3 & 6 & -3 \\ 0 & -3 & 4 \end{bmatrix}$$

The mass matrix for the damaged system is the same as that of the undamaged system but the stiffness matrix will be assumed unknown.

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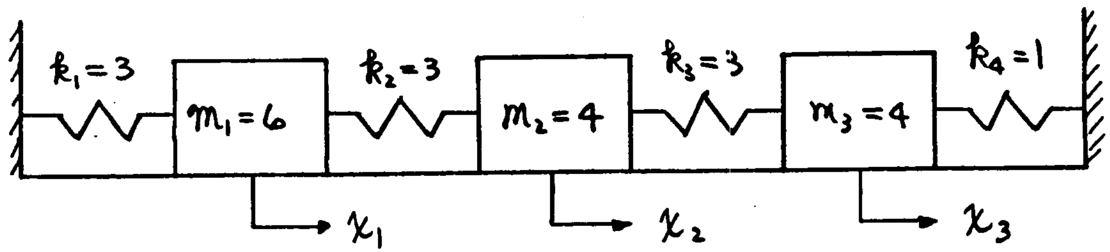
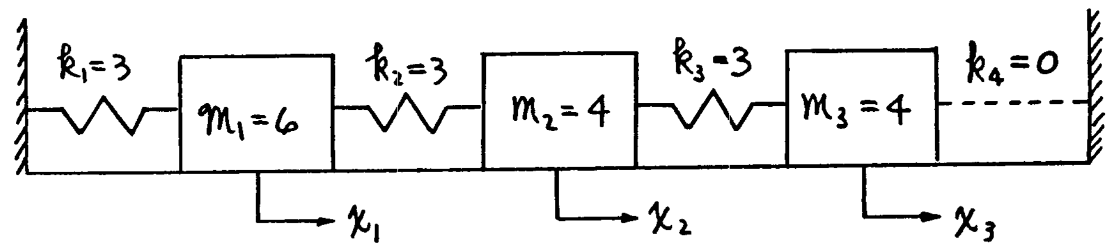


Undamaged System



Damaged System

ILLUSTRATIVE EXAMPLE

The eigenvalues and eigenvectors of both systems are shown in Table 1. The following conditions will be assumed; (1) the mathematical model of the undamaged system, i.e., the mass and stiffness distribution, are known and have been verified by test data, (2) the mass distribution and geometric configuration of the damaged system remain unchanged, and (3) the natural frequencies and mode shapes of the damaged system are measured, they may be incomplete in the sense that not all the modes are measured. It is obvious that substantial changes exist for the first mode. This is an indication that the structural system has changed and since the mass distribution for this type of damage remains the same, it must be the stiffness that has changed.

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TABLE 1

Eigenvalues and Eigenvectors

		Undamaged			Damaged		
Mode		1	2	3	1	2	3
Frequency		0.500	1.000	1.500	0.375	0.953	1.483
Mode Shape	X_1	2.000	1.000	2.000	2.184	1.101	2.312
	X_2	3.000	0	-5.000	3.756	0.200	-5.549
	X_3	3.000	-1.000	3.000	4.624	-0.944	2.870

TABLE 2

Comparisons of Frequency and Mode Shape

Mode No.	1	2	3
Frequency (%)	25.00	4.70	1.13
Mode Shape	0.271	0.232	0.083

kinetic energy distribution for the undamaged and damaged system have been calculated and are shown in Table 3. The kinetic energy distribution for each mode is listed in percentages for both the undamaged and damaged system. The comparison indicates that biggest difference between the two system is always occur at the degree-of-freedom X_3. It is reasonable to postulate that the damaged element is associated or physically attached to X_3. There are two structural elements associated with the X_3 degree-of-freedom, namely, the K_3 and K_4 springs. The K_3 spring is also associated with the X_2 degree-of-freedom whose kinetic energy remains relatively unchanged between the undamaged and damaged system for all three modes. This implies that the corresponding strain energy distribution in the elements associated with X_2 degree-of-freedom, i.e., the springs, K_2 and K_3, is relatively unchanged. Therefore, the only element whose strain energy must be changed to accommodate the kinetic energy distribution change is the K_4 spring. This concludes that the damage location is at the spring K_4.

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TABLE 3

Kinetic Energy Distribution
(%)

Mode	1		2		3	
DOF	Undamaged	Damaged	Undamaged	Damaged	Undamaged	Damaged
X_1	25.00	16.77	60.00	66.15	15.00	17.07
X_2	37.50	32.98	0	1.47	62.50	65.45
X_3	37.50	50.16	40.00	32.39	22.50	17.49

From the mode one results, the stiffness reductions for spring K_1 to K_3 are from 1.56% to 4.22%. However, for spring K_4, the stiffness reduction is 97.69%. It is obvious that the spring k_4 has suffered major damage and in view of the large stiffness reduction, one may conclude that the spring has been broken. Mode one test data correctly assesses the damage. As expected mode 2 and 3 test data provide somewhat ambiguous results because of the small differences between the data from the damaged and the undamaged systems. Although the spring K_4 is identified to be damaged by a 71.46% or 64.06% stiffness reduction, these numbers are not convincing enough to indicate a breakage. However, these numbers should be sufficient to indicate that a damage has occurred at the spring K_4.

TABLE 4

Damage Assessment for
Each Mode

Mode / Element	1		2		3	
	ΔK_j	Stiffness Reduction (%)	ΔK_j	Stiffness Reduction (%)	ΔK_j	Stiffness Reduction (%)
K_1	-.0468	1.56	.2391	-7.97 (increased)	-.4440	14.8
K_2	-.0694	2.31	-.2992	9.97	.1293	-4.31
K_3	-.1265	4.22	-.2378	7.93	-.1236	4.12
K_4	-.9769	97.69	-.7146	71.46	-.6406	64.06

It is clearly demonstrated that the proposed approach works perfectly in this case. The K_4 spring is identified as have 100% stiffness reduction which is total breakage and the springs K_1, K_2, and K_4 are identified as having less than 1% of stiffness changes. It is interesting to note when mode 2 and 3 data used separately, it provided less accurate assessment but when combined, a very accurate assessment is obtained.

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TABLE 5

Damage Assessment for Multiple Modes

Mode / Element	1 and 2		2 and 3		3 and 1		1, 2 and 3	
	ΔK_j	Stiffnes Reduction(%)	ΔK_j	Stiffness Reduction(%)	ΔK_j	Stiffness Reduction(%)	ΔK_j	Stiffness Reduction(%)
K_1	-.0003	.01	-.0038	.13	.0018	-.06 (increased)	.0010	.03 (increased)
K_2	-.0039	.13	-.0003	.01	-.0019	.06	-.0017	.06
K_3	-.0049	.16	-.0025	.08	-.0011	.04	-.0012	.04
K_4	-.9996	99.96	-.9961	99.61	-1.0000	100	-1.0000	100

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CONCLUDING REMARKS

- Theory for the detection of damage occurrence, location and extend of stiffness reduction is developed.
- Theory is tested on a simple structure and a truss type mast structure.
- Further development is required, especially the application of actual field measurement.